Riemannian Wavefield Extrapolation

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Outline

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1. Wave propagation in Continuum media

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2. OWWE and some extrapolation methods

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3. Riemannian wavefield extrapolation, steep topography, finite difference approach

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4. Objectives

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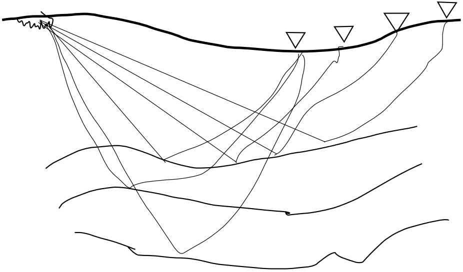
1. Wave propagation in Continuum media
2. OWWE and some extrapolation methods
3. Riemannian wavefield extrapolation, steep topography, finite difference approach
4. Objectives
5. Some results

Wave Propagation in Continuum Media (M. Slawinski, 2010 (book))



Wave Propagation in Continuum Media

- Hook's law
- Cauchy's equations of motion
- Wave equation for P-waves in homogeneous and isotropic media
- Wave equation for S-waves in homogeneous and isotropic media

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Wave Propagation in Continuum Media

- Hook's law

$$\sigma_{ij} = \sum_{k,l} \mathcal{C}_{ijkl} \; \epsilon_{kl}$$

where

$$\begin{aligned} \sigma_{ij} &: \text{ is the strain tensor,} \\ \mathcal{C}_{ijkl} &: \text{ is the stiffnes tensor,} \\ \epsilon_{kl} &: \text{ is the stress tensor.} \end{aligned}$$

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Wave Propagation in Continuum Media

- Hook‘s law
- Cauchy’s equations of motion
  From the balance of momentum one gets

$$\rho(\vec{x})\frac{\partial^2 \vec{u}_i}{\partial t^2} = \sum_j \frac{\partial}{\partial x_j}\sigma_{ij}$$

- Wave equation for P-waves in homogeneous and isotropic media
- Wave equation for S-waves in homogeneous and isotropic media

Wave Propagation in Continuum Media

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  From the balance of momentum one gets

$$\rho(\vec{x})\frac{\partial^2 \vec{u}_i}{\partial t^2} = \sum_j \frac{\partial}{\partial x_j}\sigma_{ij}$$

For an Isotropic media

$$\sigma_{ij} = \lambda\delta_{ij}\sum_k \epsilon_{kk} + 2\mu\epsilon_{ij}$$

- Wave equation for P-waves in homogeneous and isotropic media
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$$\rho(\vec{x})\frac{\partial^2 \vec{u}_i}{\partial t^2} = \sum_j \frac{\partial}{\partial x_j}\sigma_{ij}$$

then

$$\rho(\vec{x})\frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + \mu)[\bigtriangledown(\bigtriangledown \cdot \vec{u})] + \mu \bigtriangledown^2 \vec{u}$$

- Wave equation for P-waves in homogeneous and isotropic media
- Wave equation for S-waves in homogeneous and isotropic media

Wave Propagation in Continuum Media

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  From the balance of momentum one gets

$$\rho(\vec{x})\frac{\partial^2 \vec{u}_i}{\partial t^2} = \sum_j \frac{\partial}{\partial x_j}\sigma_{ij}$$

In general curvilinear coordinates

$$\bigtriangledown^2 \vec{u} = \bigtriangledown(\bigtriangledown \cdot \vec{u}) - \bigtriangledown \times (\bigtriangledown \times \vec{u})$$

and defining

$$\begin{aligned} \varphi &= \bigtriangledown \cdot \vec{u} \\ \psi &= \bigtriangledown \times \vec{u} \end{aligned}$$

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- Wave equation for S-waves in homogeneous and isotropic media

Wave Propagation in Continuum Media

- Hook's law
- Cauchy's equations of motion
  From the balance of momentum one gets

$$\rho(\vec{x})\frac{\partial^2 \vec{u}_i}{\partial t^2} = \sum_j \frac{\partial}{\partial x_j}\sigma_{ij}$$

we get

$$\rho(\vec{x})\frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu) \bigtriangledown \varphi - \mu \bigtriangledown \times \psi$$

- Wave equation for P-waves in homogeneous and isotropic media
- Wave equation for S-waves in homogeneous and isotropic media

Wave Propagation in Continuum Media

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Wave Propagation in Continuum Media

- Hook‘s law
- Cauchy’s equations of motion
- Wave equation for P-waves in homogeneous and isotropic media

$$\nabla^2\varphi - \frac{1}{v_p^2}\frac{\partial^2\varphi}{\partial t^2} = 0$$

where

$$v_p = \left(\frac{\lambda + 2\mu}{\rho}\right)^{\frac{1}{2}}$$

- Wave equation for S-waves in homogeneous and isotropic media

Wave Propagation in Continuum Media

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- Wave equation for P-waves in homogeneous and isotropic media
- Wave equation for S-waves in homogeneous and isotropic media

Wave Propagation in Continuum Media

- Hook‘s law
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$$\bigtriangledown^2\psi - \frac{1}{v_s^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

where

$$v_s = \left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}$$

On Wave equation

On Wave equation

Consider the IVP

$$
\begin{aligned}
\nabla^2 \vec{u} - \frac{1}{v^2}\frac{\partial^2 \vec{u}}{\partial t^2} &= 0 \\
\vec{u}(\vec{x}, 0) &= \gamma(\vec{x}) \\
\frac{\partial \vec{u}}{\partial t}\Big|_{t=0} &= \eta(\vec{x})
\end{aligned}
$$

On Wave equation

- In one dimension (1-D)

On Wave equation

- In one dimension (1-D)

$$u(x,t) = \frac{1}{2}\left[\gamma(x+vt) + \gamma(x-vt) + \frac{1}{v}\int_{x-vt}^{x+vt} \eta(s)ds\right]$$

where

$$\begin{aligned} \gamma(x) &= f(x) + g(x) \\ \eta(x) &= v[f'(x) + g'(x)] \end{aligned}$$

for some $f, g \in \mathcal{C}^2(\Omega)$

On Wave equation

- In one dimension (1-D)
- In two dimensions (2-D)

On Wave equation

- In one dimension (1-D)
- In two dimensions (2-D)

$$
\begin{aligned}
\vec{u}(\vec{x}, t) &= \frac{d}{dt}\left[\frac{4\pi^2}{v}\iint_{D(\vec{x},vt)} \frac{\gamma(s_1, s_2)}{\sqrt{(vt)^2 - [(s_1 - x_1)^2 + (s_2 - x_2)^2]}} ds_1 ds_2\right] \\
&+ \frac{4\pi^2}{v}\iint_{D(\vec{x},vt)} \frac{\eta(s_1, s_2)}{\sqrt{(vt)^2 - [(s_1 - x_1)^2 + (s_2 - x_2)^2]}} ds_1 ds_2
\end{aligned}
$$

OWWE. Extrapolation methods

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- Phase-shift (J.Gazdag)

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$$
\begin{aligned}
\varphi(k_x, z_j, \omega) &= \varphi(k_x, z_{j-1}, \omega) e^{ik_z \Delta z} \\
\varphi(k_x, z, \omega) &= \mathcal{F}[\psi(x, z, \omega)] \\
\varphi(k_x, z_0, \omega) &:= \text{Data}
\end{aligned}
$$

OWWE. Extrapolation methods

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- Split-Step Fourier Migration (P.L. Stoffa)

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$$
\begin{aligned}
s(\vec{r}, z) &= \frac{2}{v(\vec{r}, z)} \\
\nabla^2 \varphi + \omega^2 s^2 &= 0 \\
s(\vec{r}, z) &= s_0(z) + \triangle\, s(\vec{r}, z) \\
\nabla^2 \varphi + \omega^2 s_0^2(z) \varphi &= -S(\vec{r}, z, \omega)
\end{aligned}
$$

OWWE. Extrapolation methods

- Phase-shift (J.Gazdag)
- Split-Step Fourier Migration (P.L. Stoffa)

$$
\begin{aligned}
\frac{\partial^2}{\partial z^2} P(k_r, z, \omega) + K_{z_0}^2 P(k_r, z, \omega) &= -\hat{S}(k_r, z, \omega) \\
P_-(\vec{r}, z_{n+1}, \omega) &= P_l(\vec{r}, z_n, \triangle z, \omega) \\
&+ i\omega \int_{z_n}^{z_{n+1}} \triangle s P_l(\vec{r}, z', d_{n+1}, \omega) dz'
\end{aligned}
$$

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$$
\begin{aligned}
\left[\frac{\partial}{\partial z} + i\sqrt{A(x,\omega)}\right]\left[\frac{\partial}{\partial z} - i\sqrt{A(x,\omega)}\right]\varphi(x,z,\omega) &= 0 \\
A(x,\omega) &= \frac{\partial^2}{\partial x^2} + \frac{\omega^2}{v^2(x,z_j)} \\
s(x,z_j) &= \frac{1}{v(x,z_j)}
\end{aligned}
$$

with the extrapolators

$$
\begin{aligned}
k_z &= \sqrt{\omega^2 s^2 - k_x^2} \\
k_{z_0} &= \sqrt{\omega^2 s_0^2 - k_x^2}
\end{aligned}
$$

we get

$$
k_z = k_{z_0}\sqrt{1 - \frac{\omega^2}{k_{z_0}^2}(s_0^2 - s^2)} \tag{1}
$$

OWWE. Extrapolation methods

- Phase-shift (J.Gazdag)
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$$k_z = k_{z_0} + k_{z_0} \sum_{n=1}^{\infty} (-1)^n \binom{\frac{1}{2}}{n} \left[\left(\frac{\omega^2 s_0^2}{\omega^2 s_0^2 - k_x^2} \right) \left(\frac{s_0^2 - s^2}{s_0^2} \right) \right]^n$$

OWWE. Extrapolation methods

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$$\psi(x, z+\triangle z, \omega) = \psi(x, z, \omega) e^{ik_{z_0}\triangle z} e^{ik_{z_0}\triangle z} \sum_{n=1}^{\infty} (-1)^n \binom{\frac{1}{2}}{n} \left[\left(\frac{\omega^2 s_0^2}{\omega^2 s_0^2 - k_x^2} \right) \left(\frac{s_0^2 - s^2}{s_0^2} \right) \right.$$

$$\psi(x, z+\triangle z, \omega) = \psi(x, z, \omega) e^{ik_{z_0}\triangle z} \left\{ 1 + \sum_{n=1}^{\infty} (-1)^n \binom{\frac{1}{2}}{n} \left[\left(\frac{\omega^2 s_0^2}{\omega^2 s_0^2 - k_x^2} \right) \left(\frac{s_0^2 - s^2}{s_0^2} \right) \right] \right.$$

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  Assume that for

$$\bigtriangledown^2\vec{u} - \frac{1}{v^2(x,z)}\frac{\partial^2\vec{u}}{\partial t^2} = f(x,z,t)$$

the operator

$$A(x,z,t) = \frac{\partial^2}{\partial x^2} - \frac{1}{v^2(x,z)}\frac{\partial^2}{\partial t^2},$$

is such that $A^{\frac{1}{2}}$, and $A^{-\frac{1}{2}}$ exist.

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  The vectorial operator

$$[A] = \begin{bmatrix} 0 & 1 \\ -A & 0 \end{bmatrix}$$

  is diagonalizable

$$[A] = \mathcal{V}^{-1}\mathcal{B}\mathcal{V}$$

OWWE. Extrapolation methods

- Phase-shift (J.Gazdag)
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- Imaging Seismic Reflections Ph.D Thesis (Timotheus Op 't Root) considering the transformation

$$\vec{u} = \mathcal{V}^{-1}\vec{\mu}$$

we get the system

$$\frac{\partial \vec{\mu}}{\partial z} = \left[\mathcal{B} - \mathcal{V}\left(\frac{\partial \mathcal{V}^{-1}}{\partial z}\right)\right]\vec{\mu} - \mathcal{V}\vec{F}$$

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- Imaging Seismic Reflections Ph.D Thesis (Timotheus Op 't Root) and for depth invariant media

$$\begin{bmatrix} \frac{\partial \mu_1}{\partial z} \\ \frac{\partial \mu_2}{\partial z} \end{bmatrix} = \begin{bmatrix} i\sqrt{A}\mu_1 + \frac{i}{2}A^{-\frac{1}{2}}f \\ -i\sqrt{A}\mu_2 - \frac{i}{2}A^{-\frac{1}{2}}f \end{bmatrix}$$

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- Full-Wave-Equation depth extrapolation (K.Sandberg, G.Beylkin)

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  For the self-adjoint operator

$$\mathcal{L} = -\left(\frac{2\pi\omega}{v(x,z)}\right)^2 - D_{xx} - D_{yy}$$

Construct the spectral family (spectral projectors)

$$\begin{aligned}\mathcal{P} &= \sum_{(k:\lambda_k \leq 0)} \lambda_k P_k \\ \mathcal{PLP} &= \sum_{(k:\lambda_k \leq 0)} \lambda_k P_k\end{aligned}$$

OWWE. Extrapolation methods

- Phase-shift (J.Gazdag)
- Split-Step Fourier Migration (P.L. Stoffa)
- High Order Generalized Screen Propagator (C. Sheng, MA.Zai)
- Imaging Seismic Reflections Ph.D Thesis (Timotheus Op 't Root)
- Full-Wave-Equation depth extrapolation (K.Sandberg, G.Beylkin) reformulate the problem as

$$\begin{aligned} \hat{p}_{zz} &= \mathcal{P}\mathcal{L}\mathcal{P}\hat{p} \\ \hat{p}(x, z_n, \omega) &= q(x, z_n, \omega) \\ \hat{p}_z(x, z_n, \omega) &= q_z(x, z_n, \omega) \end{aligned}$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

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- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
  Consider the monochromatic wave equation for an acoustic wavefield

$$
\begin{aligned}
\nabla_\xi^2 \mathcal{U} &= -\omega^2 s_\xi^2 \mathcal{U}, \text{where} \\
\nabla_\xi^2 \mathcal{U} &= \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial \xi_i} \left(\sqrt{|g|} g^{ij} \frac{\partial \mathcal{U}}{\partial \xi_j} \right)
\end{aligned}
$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
  This equation can be written as

$$n^j \frac{\partial \mathcal{U}}{\partial \xi_j} + m^{ij} \frac{\partial^2 \mathcal{U}}{\partial \xi_i \partial \xi_j} = -\sqrt{|g|}\omega^2 s_\xi^2 \mathcal{U}$$

  where

$$n^j \;,\; m^{ij} \quad \text{depend on the metric.}$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
  Fourier transforming $\xi_\nu \leftrightarrow k_\nu$

  $$(m^{ij} k_{\xi_i} - in^j) k_{\xi_j} = \sqrt{|g|} \omega^2 s_\xi^2,$$

  Solving for k_{ξ_3} leads to

  $$k_{\xi_3} = -a_1 k_{\xi_1} - a_2 k_{\xi_2} + ia_3 \pm [a_4^2 \omega^2 - a_5^2 k_{\xi_1}^2 - a_6^2 k_{\xi_2}^2 - a_7 k_{\xi_1} k_{\xi_2} + ia_8 k_{\xi_1} + ia_9 k_{\xi_2} - a_{10}^2]^{1/2}$$

  and then extrapolate

  $$\mathcal{U}(\xi_3 + \triangle \xi_3, k_{\xi_1}, k_{\xi_2}, \omega) = \mathcal{U}(\xi_3, k_{\xi_1}, k_{\xi_2}, \omega) e^{ik_{\xi_3} \triangle \xi_3}$$

  Some extrapolators

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
  2D nonorthogonal coordinate system.

$$k_{\xi_3} = -a_1 k_{\xi_1} + i a_3 \pm [a_4^2 \omega^2 - a_5^2 k_{\xi_1}^2 + i a_8 k_{\xi_1} - a_{10}^2]^{1/2}$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
  2D orthogonal coordinate system.

$$k_{\xi_3} = ia_3 \pm [a_4^2\omega^2 - a_5^2 k_{\xi_1}^2 + ia_8 k_{\xi_1} - a_{10}^2]^{1/2}$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge) 3D semiorthogonal coordinate system.

$$k_{\xi_3} = ia_3 \pm [a_4^2\omega^2 - a_5^2 k_{\xi_1}^2 - a_6^2 k_{\xi_2}^2 - a_7 k_{\xi_1} k_{\xi_2} + ia_8 k_{\xi_1} + ia_9 k_{\xi_2} - a_{10}^2]^{1/2}$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
- Steep Topography (J.Shragge, P.Sava)

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
- Steep Topography (J.Shragge, P.Sava)
  Let

$$\begin{array}{lll} U & := & \text{Unit circle} \\ UHP & := & \text{Upper half plane} \\ R & := & \text{A rectangle} \end{array}$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
- Steep Topography (J.Shragge, P.Sava)

$$\begin{aligned} f &: U \to UHP \\ f(z) &= \frac{z-i}{z+i} \end{aligned}$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
- Steep Topography (J.Shragge, P.Sava)

$$
\begin{aligned}
g &: UHP \rightarrow R \\
g(k) &= \int_0^z \frac{d\zeta}{\sqrt{1-\zeta^2}\sqrt{1-k^2\zeta^2}}
\end{aligned}
$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
- Steep Topography (J.Shragge, P.Sava)
- Finite difference approach (J. Shragge)

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
- Steep Topography (J.Shragge, P.Sava)
- Finite difference approach (J. Shragge)
  Consider the acoustic wave equation

$$
\begin{aligned}
\nabla_\xi^2 \mathcal{U} - \frac{1}{v_\xi^2}\frac{\partial^2 \mathcal{U}}{\partial t^2} &= F_\xi \\
\zeta^i \frac{\partial \mathcal{U}}{\partial \xi_i} + g^{ij}\frac{\partial^2 \mathcal{U}}{\partial \xi_i \partial \xi_j} &= \frac{1}{v_\xi^2}\frac{\partial^2 \mathcal{U}}{\partial t^2} + F_\xi
\end{aligned}
$$

Riemannian wavefield extrapolation, steep topography, finite difference approach

- Riemannian wavefield extrapolation (P.Sava, S.Fomel, J.Shragge)
- Steep Topography (J.Shragge, P.Sava)
- Finite difference approach (J. Shragge)

$$\zeta^1 \frac{\partial \mathcal{U}}{\partial \xi_1} \approx \frac{\zeta^1_{l,m,n}}{\triangle\ \xi} \sum_{i=1}^{4} F_i[\mathcal{U}^p_{l+i,m,n} - \mathcal{U}^p_{l-i,m,n}]$$

$$\zeta^2 \frac{\partial \mathcal{U}}{\partial \xi_2} \approx \frac{\zeta^2_{l,m,n}}{\triangle\ \xi} \sum_{i=1}^{4} F_i[\mathcal{U}^p_{l,m+i,n} - \mathcal{U}^p_{l,m-i,n}]$$

$$\zeta^3 \frac{\partial \mathcal{U}}{\partial \xi_3} \approx \frac{\zeta^3_{l.m.n}}{\triangle\ \xi} \sum_{i=1}^{4} F_i[\mathcal{U}^p_{l,m,n+i} - \mathcal{U}^p_{l,m,n-i}]$$

$$g^{11} \frac{\partial^2 \mathcal{U}}{\partial \xi_1^2} \approx \frac{g^{11}_{l,m,n}}{\triangle\ \xi^2} \sum_{i=0}^{4} S_i[\mathcal{U}^p_{l+i,m,n} + \mathcal{U}^p_{l-i,m,n}]$$

$$g^{13} \frac{\partial^2 \mathcal{U}}{\partial \xi_1 \partial \xi_3} \approx \frac{g^{13}_{l,m,n}}{\triangle\ \xi^2} \sum_{j=1}^{2} \sum_{i=1}^{2} M_{ij}[\mathcal{U}^p_{l+i,m,n+j} + \mathcal{U}^p_{l-i,m,n-j} - \mathcal{U}^p_{l+i,m,n-j} - \mathcal{U}^p_{l-i,m,n+j}]$$

Objectives

Objectives

- Diseñar sistemas coordenados Riemannianos que transformen conforme con el campo extrapolado, el modelo de velocidad o la geometría adquirida.
-
-
-
-
-

Objectives

-
- Generalizar los operadores de extrapolación de orden superior, tales como GPSPI, NSPS, GPS, FFD, diferencias finitas; a espacios Riemannianos.

$$\nabla_{\xi}^{2}\mathcal{U} = -\omega^{2} s_{\xi}^{2}(\vec{\xi})\mathcal{U}$$

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-
-

Objectives

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- Proponer nuevos operadores, derivados de la teoría de operadores pseudodiferenciales y operadores integrales de Fourier, los cuales permitan extrapolar, en un sentido analítico, el campo de onda en medios con variaciones laterales de la velocidad, en espacios Riemannianos.
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Objectives

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- Diseñar algoritmos de migración PSPI, SSF multicoeficiente, para el campo extrapolado.
-
-

Objectives

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- Determinar descomposiciones de las soluciones a la ecuación

$$\rho \frac{\partial^2 u^i}{\partial t^2} = \lambda g^{ij} \nabla_j \nabla_k u^k + \mu g^{jk} \nabla_j \nabla_k u^i + \mu g^{ik} \nabla_j \nabla_k u^j$$

  para medios con anisotropía.
-

Objectives

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-
- Determinar relaciones funcionales entre el tensor de esfuerzos σ_{ij} y el tensor métrico g_{ij} derivadas de la ecuación de movimiento de Cauchy

$$\left\{ \frac{v^2(\vec{\xi})}{\sqrt{|g|}} \frac{\partial}{\partial \xi_j} \left[\sqrt{|g|} g^{ij} \frac{\partial \mathcal{U}}{\partial \xi_i} \right] \right\}_i = \frac{1}{\rho} f_i \left[\sum_j \frac{\partial}{\partial x_j} \sigma_{ij} \right]$$

Some results

Some results

- Rapid Expansion Method

Some results

- Rapid Expansion Method
  Consider the equation

$$
\begin{aligned}
\frac{\partial^2 U(x,t)}{\partial t^2} &= -L^2[U(x,t)] \\
U(x,0) &= U_0 \\
\frac{\partial U}{\partial t} &= \dot{U}_0
\end{aligned}
$$

  for which the solution can be written as

Some results

- Rapid Expansion Method

$$U(x, t+\triangle t) = 2U(x, t)Cos[L \triangle t] - U(x, t-\triangle t)$$

Some results

- Rapid Expansion Method
  The cosine function can be expanded in a Taylor series

$$cos[L \triangle t] = 1 - \frac{L^2(\triangle t)^2}{2!} + \frac{L^4(\triangle t)^4}{4!} \cdots$$

Some results

- Rapid Expansion Method
  or

$$
\begin{aligned}
cos[L \triangle t] &= \sum_{k=0}^{\infty} C_{2k} J_{2k}(\triangle tR) Q_{2k}\left(\frac{iL}{R}\right) \\
C_{2k} &:= \text{Constants} \\
J_{2k} &:= \text{Bessel polynomials} \\
R &= \pi \nu_{\max} \sqrt{\left(\frac{1}{\triangle x}\right)^2 + \left(\frac{1}{\triangle z}\right)^2} \\
Q_{2k} &:= \text{Chebyshev polynomials}
\end{aligned}
$$

where

$$
\begin{aligned}
Q_0(w) &= 1 \\
Q_2(w) &= 1 + 2w^2 \\
Q_{k+2}(w) &= 2(1 + 2w^2) Q_k(w) - Q_{k-2}(w)
\end{aligned}
$$

Some results

- Rapid Expansion Method
- Sheared 2D Cartesian System

Some results

- Rapid Expansion Method
- Sheared 2D Cartesian System

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & cos(\theta) \\ 0 & sen(\theta) \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_3 \end{bmatrix}$$

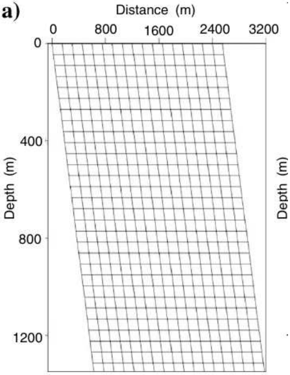
Some results

- Rapid Expansion Method
- Sheared 2D Cartesian System

$$g_{ij} = \begin{bmatrix} 1 & cos(\theta) \\ cos(\theta) & 1 \end{bmatrix}$$

Some results

- Rapid Expansion Method
- Sheared 2D Cartesian System



Some results

- Rapid Expansion Method
- Sheared 2D Cartesian System
- Polar Ellipsoidal Coordinates

Some results

- Rapid Expansion Method
- Sheared 2D Cartesian System
- Polar Ellipsoidal Coordinates

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} a(\xi_3)\xi_1 cos(\xi_3) \\ a(\xi_3)\xi_1 sen(\xi_3) \end{bmatrix}$$

Some results

- Rapid Expansion Method
- Sheared 2D Cartesian System
- Polar Ellipsoidal Coordinates

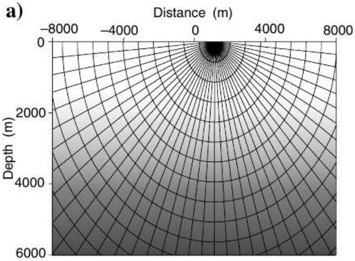
$$g_{ij} = \left[\begin{array}{cc} a^2 & \xi_1 ab \\ \xi_1 ab & \xi_1^2(a^2 + b^2) \end{array} \right]$$

where

$$\begin{aligned} a &= a(\xi_3) \\ b &= \frac{\partial a}{\partial \xi_3} \end{aligned}$$

Some results

- Rapid Expansion Method
- Sheared 2D Cartesian System
- Polar Ellipsoidal Coordinates



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